PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) From the set of vectors (1,0,1), (0,0,1) and (1,1,0) construct a set of orthonormal vectors.

    (b) State and prove the expansion theorem in linear vector space.

17. State and prove the Cauchy Residue theorem and then find the residues of $f(z) = \frac{ze^{i\theta}}{z^4 + a^4}$ at its poles.

18. Find the eigen-values and the normalized eigen vectors of the following matrices

    (a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{bmatrix}$

19. Find the Fourier transform of the Gaussian distribution function $f(x) = Ne^{-ax^2}$ Where N and a are constants.

20. Prove that

    (a) $e^{2zx - z^2} = \sum_{n=0}^{n=\infty} \frac{H_n(x)}{n!} z^n$

    (b) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

APRIL/MAY 2024

23PPH11 — MATHEMATICAL PHYSICS

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Subspaces of vector spaces.

2. Show that the vectors (1,2,–3), (1,3,–2) and (2,–1,5) are linearly independent.

3. What is singular point of analytic function?

4. Find the poles for the function $f(z) = \frac{z}{\text{Cos}\, z}$.

5. Find the inverse of the matrix for $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$.

6. Show that the eigen values of a Hermitian matrix are real.

7. State linearity property of Laplace transform.

8. Expand the function $f(x) = \sin x$ as a cosine series in the interval $(0, \pi)$.

9. If $P_n(x)$ is a Legendre polynomial, then value of $\int_{-1}^{+1} [p_n(x)]^2 dx$ is.

10. Define Generating function.

PART B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Describe the Schmidt's orthogonalization process in some n-dimensional vector space.

Or

(b) Obtain an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by $X_1 = (1,0,1,0)$, $X_2 = (1,1,1,1)$, $X_3 = (-1,2,0,1)$.

12. (a) Show that the real and imaginary parts of the function log z satisfy the Cauchy-Reimann equations, when z is not zero.

Or

(b) Expand the function as a Taylor's series $f(z) = \dfrac{1}{z+1}$ about $z = 1$.

13. (a) Find a similarity transformation that diagonalises the matrix A given by $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

Or

(b) If $A = \dfrac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, Show that $A^{-1} = A^T, A^T$ being a transpose of matrix A.

14. (a) Prove the similarity theorem or change of scale property from $g(\omega)$ is the Fourier transform of $f(t)$, the Fourier transform of $f(at)$ is $\dfrac{1}{a} g\left(\dfrac{\omega}{a}\right)$.

Or

(b) Find the inverse Laplace transform of In $\left(\dfrac{s^2 + w^2}{s^2}\right)$.

15. (a) Derive the orthogonality relation $\int_{-1}^{+1} p_n(x) p_m(x) dx = 0$ if $m \neq n$.

Or

(b) Find the derivatives of Dirac delta function at the origin $x = 0$.